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Theoretical constraints on the couplings of non-exotic minimal Z' bosons

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Abstract

We have combined perturbative unitarity and renormalisation group equation arguments in order to find a dynamical way to constrain the space of the gauge couplings $(g'_1, \widetilde{g})$ of the so-called "Minimal Z' Models". We have analysed the role of the gauge couplings evolution in the perturbative stability of the two-to-two body scattering amplitudes of the vector and scalar sectors of these models and we have shown that perturbative unitarity imposes an upper bound that is generally stronger than the triviality constraint. We have also demonstrated how this method quantitatively refines the usual triviality bound in the case of benchmark scenarios such as the $U(1)_\chi$, the $U(1)_R$ or the "pure" $U(1)_{B-L}$ extension of the Standard Model. Finally, a description of the underlying model structure in Feynman gauge is provided.

I. INTRODUCTION

Nowadays the phenomenological importance of Beyond the Standard Model (BSM) physics at the TeV scale is recognised by the global experimental effort at the Large Hadron Collider (LHC).

It is common belief that a Z' boson is among the first new objects that can potentially be detected at the LHC. The existing extensive literature is testimonial to the growing interest in them (see e.g. [1–4]). A particularly interesting class of theoretical scenarios incorporating a Z' boson are the so-called "(non-exotic/non-anomalous) Minimal Z' Models", extensively studied in the recent years [5–10].

These models are based on an extension of the Standard Model (SM) gauge group with a further $U(1)$ symmetry factor. The anomaly cancellation conditions imply the inclusion of three generations of right-handed neutrinos in the fermion sector, while the breaking of the new gauge group is provided by an extra singlet Higgs boson (thereby making the Z' boson a massive state).

The purpose of this paper is to show that renormalisation group equation (RGE) based techniques [11–15] as well as a standard perturbative unitarity criterion [16] can be combined to give a dynamical way to constrain the two gauge couplings (g'_1 and $\widetilde{g}$) of a set of Minimal Z' Models, with a particular attention devoted to some benchmark scenarios such as the "minimal" $U(1)_{B-L}$, the $U(1)_R$ (no fermion charge associated to the left-handed fermions) and the $SO(10)$-inspired $U(1)_\chi$ extensions (see [4] for an extensive overview).

To this end, we propose a detailed study of the Goldstone and Higgs sectors of this kind of models with a view to extract the most stringent bounds on the (evolving) gauge couplings. We will make a comparison between this method and triviality arguments, showing that calling for perturbative unitarity stability conditions gives stronger constraints on g'_1 and $\widetilde{g}$ with respect to traditional triviality assumptions over most of the parameter space. For an exhaustive description of the theoretical setup and of our conventions see [17], where also the RGE equations can be found. Finally, regarding perturbative unitarity techniques, we will expand below upon the methodology outlined in [18].

This work is organised as follows: in Section II we introduce our parametrisation of the Scalar Lagrangian of the Minimal Z' Models, in Section III we describe the theoretical methods adopted to constrain the gauge couplings, in Section IV we present our numerical

results while in the last section we give our conclusions; in Appendix A we discuss the gauge-fixing Lagrangian of Minimal Z' Models, in Appendix B we list the set of Feynman rules that is relevant in our calculation, and in Appendix C we give some explicit analytical results that have been used in this paper.

II. THE PARAMETRISATION OF MINIMAL Z' MODELS

We describe here our parametrisation of the Minimal Z' Models. Following [17], the SM gauge group is augmented by a $U(1)$ factor, related to the Baryon minus Lepton $(B-L)$ gauged number. In the complete model, the classical gauge invariant Lagrangian, obeying the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ gauge symmetry, can be decomposed as:

$$\mathscr{L} = \mathscr{L}_s + \mathscr{L}_{YM} + \mathscr{L}_f + \mathscr{L}_Y \,. \tag{1}$$

In this paper we are mainly interested in the scalar part of the Lagrangian

$$\mathscr{L}_s = \left(D^{\mu} H\right)^{\dagger} D_{\mu} H + \left(D^{\mu} \chi\right)^{\dagger} D_{\mu} \chi - V(H,\chi) \,, \tag{2}$$

with the scalar potential given by

$$\begin{aligned} V(H,\chi) \;&=\; m^2 H^{\dagger} H + \mu^2 \mid \chi \mid^2 + \begin{pmatrix} H^{\dagger} H & \mid \chi \mid^2 \end{pmatrix} \begin{pmatrix} \lambda_1 & \frac{\lambda_3}{2} \\ \frac{\lambda_3}{2} & \lambda_2 \end{pmatrix} \begin{pmatrix} H^{\dagger} H \\ \mid \chi \mid^2 \end{pmatrix} \\ &=\; m^2 H^{\dagger} H + \mu^2 \mid \chi \mid^2 + \lambda_1 (H^{\dagger} H)^2 + \lambda_2 \mid \chi \mid^4 + \lambda_3 H^{\dagger} H \mid \chi \mid^2 \,, \end{aligned} \tag{3}$$

where H and χ are the complex scalar Higgs doublet and singlet fields, respectively.

We generalise the SM discussion of spontaneous Electro-Weak Symmetry Breaking ($EWSB$) to the more complicated classical potential of equation (3). To determine the conditions for $V(H,\chi)$ to be bounded from below, it is sufficient to study its behaviour for large field values, controlled by the matrix in the first line of equation (3). Requiring such a matrix to be positive-definite, we obtain the conditions:

$$4\lambda_1 \lambda_2 - \lambda_3^2 > 0 \,, \tag{4}$$

$$\lambda_1, \lambda_2 > 0 \,. \tag{5}$$

If the above conditions are satisfied, we can proceed to the minimisation of V as a function of constant Vacuum Expectation Values ($VEVs$) for the two Higgs fields. In the Feynman gauge, we can parametrise the scalar fields as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} -i(\phi_1 - i\phi_2) \\ v + (h + i\phi_3) \end{pmatrix}, \qquad \chi = \frac{1}{\sqrt{2}} \left(x + (h' + i\phi_4) \right), \tag{6}$$

where $w^{\pm} = \phi_1 \mp i\phi_2$ are the would-be Goldstone bosons of $W^{\pm}$, while ϕ_3 and ϕ_4 will mix to give z and z', the would-be Goldstone bosons of the Z and Z' bosons, respectively. The real and non-negative $VEVs$ are v and x, for the Higgs doublet and singlet, respectively.

We denote by h_1 and h_2 the scalar fields of definite masses, m_{h_1} and m_{h_2} respectively, and we conventionally choose $m^2_{h_1} < m^2_{h_2}$. After standard manipulations, the explicit expressions for the scalar mass eigenvalues and eigenvectors are:

$$m^2_{h_1} = \lambda_1 v^2 + \lambda_2 x^2 - \sqrt{(\lambda_1 v^2 - \lambda_2 x^2)^2 + (\lambda_3 x v)^2}, \tag{7}$$

$$m^2_{h_2} = \lambda_1 v^2 + \lambda_2 x^2 + \sqrt{(\lambda_1 v^2 - \lambda_2 x^2)^2 + (\lambda_3 x v)^2}, \tag{8}$$

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h \\ h' \end{pmatrix}, \tag{9}$$

where $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$ fulfils[1]:

$$\sin 2\alpha = \frac{\lambda_3 x v}{\sqrt{(\lambda_1 v^2 - \lambda_2 x^2)^2 + (\lambda_3 x v)^2}}, \tag{10}$$

$$\cos 2\alpha = \frac{\lambda_1 v^2 - \lambda_2 x^2}{\sqrt{(\lambda_1 v^2 - \lambda_2 x^2)^2 + (\lambda_3 x v)^2}}. \tag{11}$$

For our numerical study of the extended Higgs sector, it is useful to invert equations (7), (8) and (10) to extract the parameters in the Lagrangian in terms of the physical quantities m_{h_1}, m_{h_2} and α:

$$\begin{aligned} \lambda_1 &= \frac{m^2_{h_2}}{4v^2}(1 - \cos 2\alpha) + \frac{m^2_{h_1}}{4v^2}(1 + \cos 2\alpha), \\ \lambda_2 &= \frac{m^2_{h_1}}{4x^2}(1 - \cos 2\alpha) + \frac{m^2_{h_2}}{4x^2}(1 + \cos 2\alpha), \\ \lambda_3 &= \sin 2\alpha \left(\frac{m^2_{h_2} - m^2_{h_1}}{2xv} \right). \end{aligned} \tag{12}$$

[1] In all generality, the whole interval $0 \leq \alpha < 2\pi$ is halved because an orthogonal transformation is invariant under $\alpha \to \alpha + \pi$. We could re-halve the interval by noting that it is invariant also under $\alpha \to -\alpha$ if we permit the eigenvalues inversion, but this is forbidden by our convention $m^2_{h_1} < m^2_{h_2}$. Thus α and $-\alpha$ are independent solutions.

In order to determine the covariant derivative, we must introduce $\mathscr{L}_{YM}$, in which the the non-Abelian field strengths therein are the same as in the SM whereas the Abelian ones can be written as follows:

$$\mathscr{L}_{YM}^{\rm Abel} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}F'^{\mu\nu}F'_{\mu\nu} \,, \tag{13}$$

where

$$F_{\mu\nu} \;=\; \partial_\mu B_\nu - \partial_\nu B_\mu \,, \tag{14}$$

$$F'_{\mu\nu} \;=\; \partial_\mu B'_\nu - \partial_\nu B'_\mu \,. \tag{15}$$

In this field basis, the covariant derivative is:

$$D_\mu \equiv \partial_\mu + ig_S T^\alpha G_\mu^{\ \alpha} + igT^a W_\mu^{\ a} + ig_1 Y B_\mu + i(\widetilde{g}Y + g'_1 Y_{B-L})B'_\mu \,. \tag{16}$$

To determine the boson spectrum, we have to expand the scalar kinetic terms like for the SM. As for the gauge bosons, we expect that there exists a mass-less gauge boson, the photon, whilst the other gauge bosons become massive. The extension we are studying is in the Abelian sector of the SM gauge group, so that the charged gauge bosons $W^\pm$ will have masses given by their usual SM expressions, being related to the $SU(2)_L$ factor only. The gauge boson spectrum is then extracted from the kinetic terms in equation (2):

$$\begin{aligned}
\left(D^\mu H\right)^\dagger D_\mu H\Big|_{gauge} \;&=\; \frac{1}{2}\partial^\mu h \partial_\mu h + \frac{1}{8}(h+v)^2 \big(0\ 1\big)\Big[gW_a^{\ \mu}\sigma_a + g_1 B^\mu + \widetilde{g}B'^\mu\Big]^2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\
&=\; \frac{1}{2}\partial^\mu h \partial_\mu h + \frac{1}{8}(h+v)^2 \Big[g^2 \left|W_1^{\ \mu} - iW_2^{\ \mu}\right|^2 \\
&\qquad\qquad + \left(gW_3^{\ \mu} - g_1 B^\mu - \widetilde{g}B'^\mu\right)^2\Big] \,,
\end{aligned} \tag{17}$$

and

$$\left(D^\mu \chi\right)^\dagger D_\mu \chi\Big|_{gauge} \;=\; \frac{1}{2}\partial^\mu h' \partial_\mu h' + \frac{1}{2}(h'+x)^2 (g'_1 2B'^\mu)^2 \,, \tag{18}$$

where we have taken $Y_\chi^{B-L} = 2$ in order to guarantee the gauge invariance of the Yukawa terms (see [19, 20] for details). In equation (17) we can recognise the SM charged gauge bosons $W^\pm$, with $M_W = gv/2$ as in the SM. The other gauge boson masses are not so simple to identify, because of mixing. In fact, in analogy with the SM, the fields of definite

mass are linear combinations of B^μ, W_3^μ and B'^μ. The explicit expressions are:

$$\begin{pmatrix} B^\mu \\ {W_3}^\mu \\ B'^\mu \end{pmatrix} = \begin{pmatrix} \cos\vartheta_w & -\sin\vartheta_w\cos\vartheta' & \sin\vartheta_w\sin\vartheta' \\ \sin\vartheta_w & \cos\vartheta_w\cos\vartheta' & -\cos\vartheta_w\sin\vartheta' \\ 0 & \sin\vartheta' & \cos\vartheta' \end{pmatrix} \begin{pmatrix} A^\mu \\ Z^\mu \\ Z'^\mu \end{pmatrix}, \tag{19}$$

with $-\frac{\pi}{4} \leq \vartheta' \leq \frac{\pi}{4}$, such that:

$$\tan 2\vartheta' = \frac{2\widetilde{g}\sqrt{g^2+g_1^2}}{\widetilde{g}^2 + 16(\frac{x}{v})^2 g_1'^2 - g^2 - g_1^2} \tag{20}$$

and

$$\begin{aligned} M_A &= 0\,, \\ M^2_{Z,Z'} &= \frac{1}{8}\left(Cv^2 \mp \sqrt{-D + v^4C^2}\right), \end{aligned} \tag{21}$$

where

$$C = g^2 + g_1^2 + \widetilde{g}^2 + 16\left(\frac{x}{v}\right)^2 g_1'^2\,, \tag{22}$$

$$D = 64v^2x^2(g^2+g_1^2)g_1'^2\,. \tag{23}$$

As for the Goldstone boson spectrum, it is possible to find a convenient way to write the mass matrix. Being $H \sim (1,2,\frac{1}{2},0)$ and $\chi \sim (1,1,0,2)$ the Higgs representations associated to each gauge group, in the gauge-Goldstone[2] bosons basis we find the following representation of the co-variant derivative:

$$\mathcal{D} = \begin{pmatrix} \frac{v}{2}g & 0 & 0 & 0 \\ 0 & \frac{v}{2}g & 0 & 0 \\ 0 & 0 & \frac{v}{2}g & 0 \\ 0 & 0 & -\frac{v}{2}g_1 & 0 \\ 0 & 0 & -\frac{v}{2}\widetilde{g} & -2xg_1' \end{pmatrix}. \tag{24}$$

In the t'Hooft-Feynman gauge, it can be verified that the vector boson mass matrix is given by $m_V^2 = \mathcal{D}(\mathcal{D})^T$. The related Goldstones mass matrix can as well be evaluated as

$$m_v^2 = (\mathcal{D})^T\mathcal{D}\,, \tag{25}$$

[2] The 5×4 matrix follows from the five gauge bosons $W^i|_{i=1,3}$, Z, Z' and the four Goldstone bosons $\phi^i|_{i=1,4}$.

therefore we get

$$m_v^2 = \begin{pmatrix} \frac{v^2}{4}g^2 & 0 & 0 & 0 \\ 0 & \frac{v^2}{4}g^2 & 0 & 0 \\ 0 & 0 & \frac{v^2}{4}(g^2+g1^2+\widetilde{g}^2) & xv\widetilde{g}g_1' \\ 0 & 0 & xv\widetilde{g}g_1' & 4x^2(g_1')^2 \end{pmatrix} . \tag{26}$$

The mass matrix in equation (26) shows that the Goldstones of the W-boson have a mass that is equivalent to the SM one, while the ϕ_3 and ϕ_4 fields mix, as it happens for the related gauge bosons. We can diagonalise the neutral Goldstone block by means of a rotation of angle α_g, defined by:

$$\tan 2\alpha_g = \frac{-8\frac{x}{v}\widetilde{g}\,g_1'}{g^2+g_1^2+\widetilde{g}^2-16\left(\frac{x}{v}g_1'\right)^2} , \tag{27}$$

obtaining, as expected, the neutral gauge boson masses as eigenvalues of the neutral Goldstone boson sub-matrix. As for the neutral gauge boson sector, the Goldstones mix only if $\widetilde{g} \neq 0$. Finally, the neutral Goldstone bosons fulfil

$$\begin{pmatrix} z \\ z' \end{pmatrix} = \begin{pmatrix} \cos\alpha_g & \sin\alpha_g \\ -\sin\alpha_g & \cos\alpha_g \end{pmatrix} \begin{pmatrix} \phi_3 \\ \phi_4 \end{pmatrix} . \tag{28}$$

Now that the scalar Lagrangian has been presented in the Feynman gauge, we have all the elements to carry on with our analysis. Although not relevant for the latter, we also present for completeness the gauge-fixing Lagrangian in Appendix A.

The generic model that has been previously introduced spans over a continuous set of minimal $U(1)$ extensions of the SM, that can be labelled by the properties of the charge assignments to the particle content.

As any other parameter in the Lagrangian, $\widetilde{g}$ and g_1' are running parameters, therefore their values have to be set at some scale. A discrete set of popular Z' models (see, e.g. [4, 5]) can be recovered by a suitable definition of both $\widetilde{g}$ and g_1'.

Even though we present results in the generic (g_1'–$\widetilde{g}$) space, we will comment on a subset of particular interest: the "pure" $B-L$ extension $U(1)_{B-L}$ ($\widetilde{g}_{EW}=0$) has a vanishing mixing between the massive neutral gauge bosons at tree-level at the EW scale, the $SO(10)$-inspired extension $U(1)_\chi$ ($\widetilde{g}_{EW}=-4/5g_1'$) preserves the mixing ratio at any energy scale and the R minimal extension $U(1)_R$ ($\widetilde{g}_{EW}=-2g_1'$) in which a Z' is coupled to right-handed fermions only. In Table I we summarise these models emerging from our parametrisation.

Model	Parametrisation
$U(1)_{B-L}$	$\widetilde{g}_{EW} = 0$
$U(1)_\chi$	$\widetilde{g}_{EW} = -4/5 g'_1$
$U(1)_R$	$\widetilde{g}_{EW} = -2 g'_1$

TABLE I: Specific parametrisations of the Minimal Z' Models: $U(1)_{B-L}$, $U(1)_\chi$ and $U(1)_R$.

III. CONSTRAINING THE $g'_1 - \tilde{g}$ SPACE

Since it has been proven that perturbative unitarity violation at high energy occurs only in vector and Higgs boson elastic scatterings, our interest is focused on the corresponding sectors that have been already presented in Section II.

Following the Becchi-Rouet-Stora (BRS) invariance (see [21]), the amplitude for emission or absorption of a "scalarly" polarised gauge boson becomes equal to the amplitude for emission or absorption of the related would-be-Goldstone boson, and, in the high energy limit ($s \gg m^2_{W^\pm,Z,Z'}$), the amplitude involving the (physical) longitudinal polarisation (the dominant one at high energies) of gauge bosons approaches the (unphysical) scalar one, proving the so-called Equivalence Theorem (ET), see [22]. Therefore, the analysis of the perturbative unitarity of two-to-two particle scatterings in the gauge sector can be performed, in the high energy limit, by exploiting the Goldstone sector instead (further details of this formalism can be found in [18]).

Moreover, since we want to focus on g'_1 and $\widetilde{g}$ limits, we assume that the two Higgs bosons of the model have masses such that no significant contribution to the spherical partial wave amplitude (see below) will come from the scalar four-point and three-point functions (that is $m_{1,2} \ll 700$ according to [18]), i.e. the Higgs masses are well below the Lee-Quigg-Tacker (LQT) limit [16]. It is important to remark that relatively high values of the Higgs masses, far below the unitarity limit, tend to lead to quartic coupling to values that become non-perturbative at high scales. On a side, this could considerably refine the unitarity bounds. On the other side, it could be non-consistent by triviality arguments (as a general rule, the larger the cut-off, the smaller the acceptable value of the Higgs mass). Beyond any doubt, given a cut-off energy, a good choice for the Higgs masses is the one explored in [17]. With this choice we exclude any other source of unitarity violation different from the size of the

g_1' and $\widetilde{g}$ gauge couplings.

Firstly, we focus on the techniques that we have used to obtain the aforementioned unitarity bounds in combination with the RGE analysis: for this, it is crucial to define the evolution of the gauge couplings via the $RGEs$ and their boundary conditions. As already established in [17, 23], the $RGEs$ of g, g_1, g_1' and $\widetilde{g}$ are:

$$
\begin{aligned}
\frac{d(g)}{d(\log\Lambda)} &= \frac{1}{16\pi^2}\left[-\frac{19}{6}g^3\right], \\
\frac{d(g_1)}{d(\log\Lambda)} &= \frac{1}{16\pi^2}\left[\frac{41}{6}g_1^3\right], \\
\frac{d(g_1')}{d(\log\Lambda)} &= \frac{1}{16\pi^2}\left[12g_1'^3 + 2\frac{16}{3}g_1'^2\widetilde{g} + \frac{41}{6}g_1'\widetilde{g}^2\right], \\
\frac{d(\widetilde{g})}{d(\log\Lambda)} &= \frac{1}{16\pi^2}\left[\frac{41}{6}\widetilde{g}\,(\widetilde{g}^2 + 2g_1^2) + 2\frac{16}{3}g_1'(\widetilde{g}^2 + g_1^2) + 12g_1'^2\widetilde{g}\right],
\end{aligned}
\tag{29}
$$

where $g(EW) \simeq 0.65$ and $g_1(EW) \simeq 0.36$. This fully fixes the evolution of g_1' and $\widetilde{g}$ with the energy scale Λ.

In the search for the maximum $g_1'(EW)$ and $\widetilde{g}(EW)$ values allowed by theoretical constraints, the contour condition

$$
g_1'(\Lambda), \widetilde{g}(\Lambda) \le k, \tag{30}
$$

also known as the triviality condition, is the assumption that enables one to solve the above system of equations and gives the traditional upper bound on the g_1'-$\widetilde{g}(EW)$ space at the EW scale.

It is usually assumed either $k = 1$ or $k = \sqrt{4\pi}$, calling for a coupling that preserves the perturbative convergence of the theory. Nevertheless, we stress again that this is an "ad hoc" assumption. Our aim, instead, is to extract the boundary conditions by perturbative unitarity arguments, showing that, under certain conditions, it represents a stronger constraint on most of the gauge couplings parameter space. For this, we exploit the theoretical techniques that are related with the perturbative unitarity analysis, since they can be used to provide constraints on the theory, with a procedure that is not far from the one firstly described in detail by [16].

A well known result is that, by evaluating the tree-level scattering amplitude of longitudinally polarised vector bosons, one finds that the latter grows with the energy of the process and, in order to preserve unitarity, it is necessary to include some other (model dependent)

interactions (for example, in the SM one needs to add the Higgs boson) and these must fulfil the unitarity criterion (again in the SM, the Higgs boson must have a mass bounded from above by the LQT limit [24]).

As already intimated, we also know that the ET allows one to compute the amplitude of any process with external longitudinal vector bosons V_L ($V = W^\pm, Z, Z'$), in the limit $m_V^2 \ll s$, by substituting each one of these with the related Goldstone boson $v = w^\pm, z, z'$, and its general validity has been proven in [22]. Schematically, if we consider a process with four longitudinal vector bosons, we have that $M(V_L V_L \to V_L V_L) = M(vv \to vv) + O(m_V^2/s)$.

While in the search for the Higgs boson mass bound it is widely accepted to assume small values for the gauge couplings and large Higgs boson masses, for our purpose we reverse such argument with the same logic: we assume that the Higgs boson masses are compatible with the unitarity limits and we study the two-to-two scattering amplitudes of the whole scalar sector, pushing the size of g_1' and $\widetilde{g}$ up to the unitarisation limit.

This limit is a consequence of the following argument: given a tree-level scattering amplitude between two spin-0 particles, $M(s, \theta)$, where θ is the scattering (polar) angle, we know that the partial wave amplitude with angular momentum J is given by

$$a_J = \frac{1}{32\pi} \int_{-1}^{1} d(\cos\theta) P_J(\cos\theta) M(s, \theta), \tag{31}$$

where P_J are Legendre polynomials, and it has been proven (see [24]) that, in order to preserve unitarity, each partial wave must be bounded by the condition

$$|\mathrm{Re}(a_J(s))| \leq \frac{1}{2}. \tag{32}$$

By direct computation, it turns out that only $J = 0$ (corresponding to the spherical partial wave contribution) leads to some bound, so we will not discuss the higher partial waves any further.

Assuming that the Higgs boson masses do not play any role in the perturbative unitarity violation, we have verified that the only divergent contribution to the spherical amplitude is due to the size of the g_1' and $\widetilde{g}$ couplings in the t-channel intermediate Z and Z' vector boson exchange contributions. In Appendix B we list the relevant 3-point Feynman rules that connect any of the two (external) scalars with either a Z or Z' (mediator). Hence, the relevant channels are represented by the 6-dimensional (symmetric) scattering matrix in Table II plus the decoupled eigenchannel $w^+w^- \to w^+w^-$.

	$z\,z$	$z\,z'$	$z'z'$	h_1h_1	h_1h_2	h_2h_2
$z\,z$	0	0	0	$\sim$	$\sim$	$\sim$
$z\,z'$	0	0	0	$\sim$	$\sim$	$\sim$
$z'z'$	0	0	0	$\sim$	$\sim$	$\sim$
h_1h_1	$\sim$	$\sim$	$\sim$	0	0	0
h_1h_2	$\sim$	$\sim$	$\sim$	0	0	0
h_2h_2	$\sim$	$\sim$	$\sim$	0	0	0

TABLE II: Scattering matrix: we have used the simbol $\sim$ just for illustrating the presence of a non-zero element in the correspondent scattering channels.

After explicit evaluation, the spherical amplitude of the decoupled w^+w^- eigenchannel, in the high energy limit, is:

$$a_{w^+w^-} = \left\{ \frac{f^z_{w^+w^-}}{16\pi} \left[1 + 4\log\left(\frac{M_Z}{\Lambda}\right)\right] + \frac{f^{z'}_{w^+w^-}}{16\pi} \left[1 + 4\log\left(\frac{M_{Z'}}{\Lambda}\right)\right] \right\}, \tag{33}$$

and each element of the scattering matrix presents the following structure:

$$a_{ij} = S_i S_j \left\{ \frac{f^z_{i,j}}{16\pi} \left[1 + 4\log\left(\frac{M_Z}{\Lambda}\right)\right] + \frac{f^{z'}_{i,j}}{16\pi} \left[1 + 4\log\left(\frac{M_{Z'}}{\Lambda}\right)\right] \right\}, \tag{34}$$

where S is a symmetry factor that becomes $1/\sqrt{2}$ if the (initial or final) state has identical particles, 1 otherwise, and Λ represents the scale of energy at which the scattering is consistent with perturbative unitarity, i.e. it is the evolution energy scale cut-off. It is important to notice that the masses of the Z and Z' act as a natural regulator that preserves both the amplitude and the spherical partial wave from any t-channel collinear divergence and that both of them are completely defined by the choice of the gauge couplings and $VEVs$ (see equation (21)). The non-vanishing coefficients of equations (33)-(34) are listed in Appendix C.

It is well-known[3] that the most stringent unitarity bounds on the g'_1-$\widetilde{g}$ space are derived from the requirement that the magnitude of the largest eigenvalue of the scattering matrix does not exceed 1/2.

[3] The diagonalisation of the scattering matrix usually leads to stronger bounds not only in the SM-case but also in BSM scenarios (e.g. [25]).

Finally, if we consider the contour of this inequality, we find exactly the boundary conditions that solve the set of differential equations in (29), giving us the upper limits for g_1' and $\widetilde{g}$ at the EW scale. In the next section we will combine all these elements to present a numerical analysis of the allowed domain of the gauge couplings.

IV. RESULTS

The set of differential equations (29) has been integrated with the well-known Runge-Kutta algorithm and both the unitarity (equation (32)) and triviality (equation (30)) conditions have been imposed as a two-point boundary value with a simple shooting method, that consisted in varying the initial gauge coupling values in dichotomous-converging steps until the bounds were fulfilled.

Apart from the gauge couplings, other parameters play a role in the computation: the $VEVs$ have been chosen in such a way that both M_Z and $M_{Z'}$ are within the allowed experimental range (see [26] and [27], respectively), and further that $M_{Z'}$ lies in the $\mathcal{O}(1-10)$ TeV range, so that it does not spoil the high energy approximation $M_{Z'} \ll \Lambda$. By direct computation we verified that the Higgs mixing angle α does not play any significant role in the analysis, hence for each analysed point of the gauge couplings parameter space we have averaged the spherical wave greatest eigenvalue over the range $-1 < \sin\alpha < +1$, finding a standard deviation never greater than $\sim 2\%$. Finally, for illustrative purposes, we have chosen the triviality condition to be fixed by $k = 1$.

As initial step of our numerical analysis, we have verified by direct computation that the spherical wave associated to the decoupled eigenchannel $w^+w^- \to w^+w^-$ gives always a negligible contribution with respect to the greatest eigenvalue of the spherical wave scattering matrix in Table II. Therefore, in Figure 1 we have overlapped the contour plots of both the greatest eigenvalue of the spherical wave scattering matrix allowed by unitarity and the gauge couplings allowed by triviality in the g_1'-$\widetilde{g}$ plane for the following values of the evolution/cut-off energy: $\Lambda = 10^{11}$ GeV (Figure 1a), $\Lambda = 10^{15}$ GeV (Figure 1b), $\Lambda = 10^{19}$ GeV (Figure 1c). It is clear that the boundary condition imposed by the perturbative unitarity stability (dashed lines) constrains the parameter space considerably more than the well-known triviality bound (dotted lines). In few cases the triviality bound is (slightly) more important than the unitarity bound: this condition is realised at energies $\ll 10^{19}$

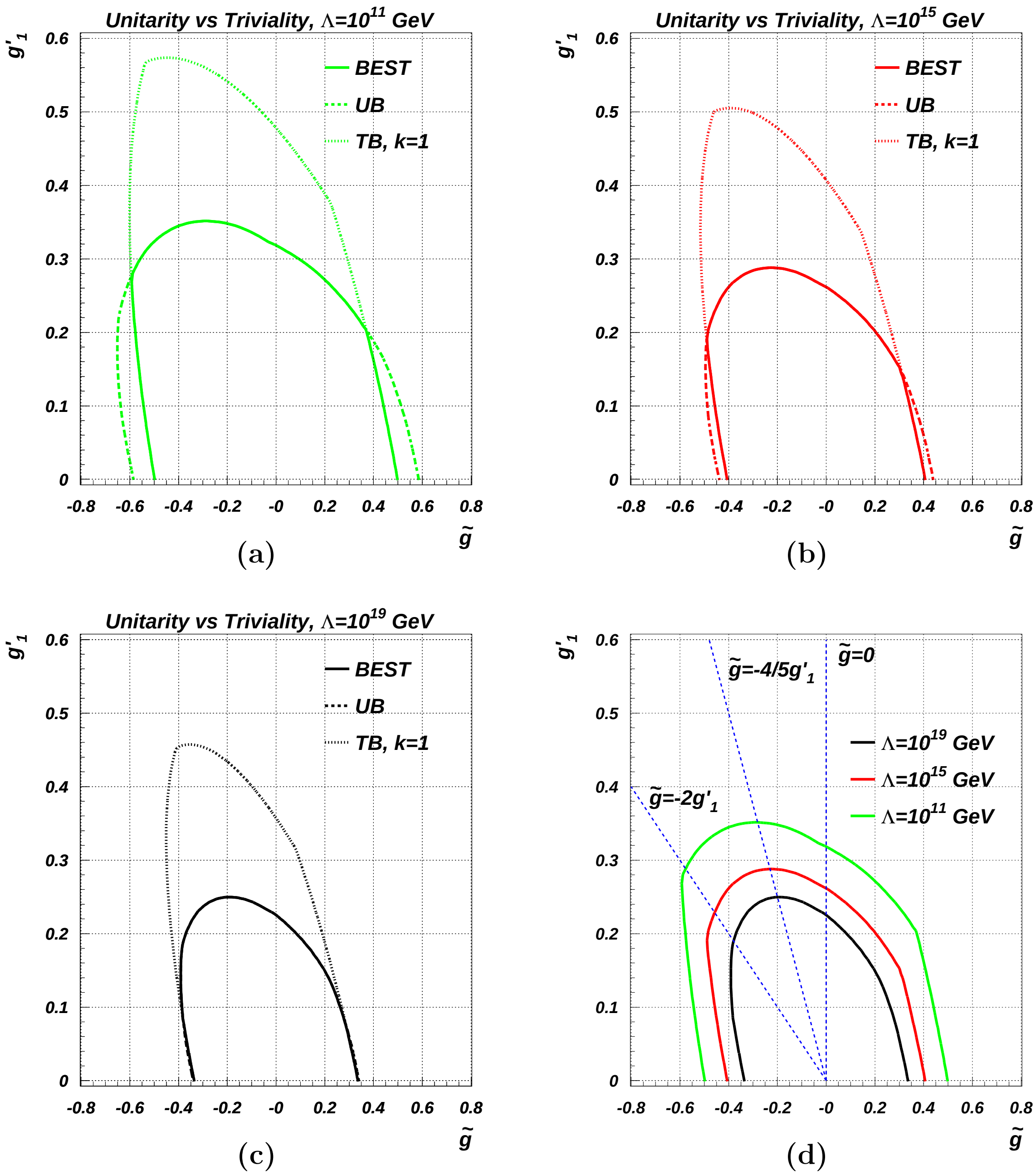

FIG. 1: Contour plot of both the greater eigenvalue of the spherical wave scattering matrix (Table II) allowed by unitarity (dashed lines) and the gauge couplings allowed by triviality (dotted lines) for several values of the cut-off energy: $\Lambda = 10^{11}$ GeV (light green/grey lines, Figure 1a), $\Lambda = 10^{15}$ GeV (dark red/grey lines, Figure 1b), $\Lambda = 10^{19}$ GeV (black lines, Figure 1c). Figure 1d shows a summary of the most stringent bounds at different values of the cut-off energy, with focus on some peculiar parametrisation of $\widetilde{g}$ (Table I).

$Log_{10}(\Lambda/\mathrm{GeV})$		7	9	11	13	15	17	19
$U(1)_{B-L}$	T	0.594	0.527	0.477	0.439	0.407	0.380	0.357
	U	0.487	0.390	0.335	0.297	0.269	0.247	0.229
$U(1)_\chi$	T	0.682	0.620	0.573	0.535	0.504	0.479	0.457
	U	0.531	0.424	0.364	0.324	0.295	0.272	0.254
$U(1)_R$	T	0.362	0.328	0.300	0.276	0.254	0.235	0.218
	U	0.429	0.344	0.293	0.258	0.232	0.210	0.192

TABLE III: Triviality bounds (with $k = 1$) and unitarity bounds on g_1' in (non-exotic) Minimal Z' Models, for several values of the energy scale Λ.

GeV and $\widetilde{g} = hg_1'$ where $|h| > 2$ (see Figure 1a-1b). Otherwise, the unitarity condition noticeably refines the bounds on the allowed parameter space considerably, as it is clear from Figure 1d, in which the most stringent bounds are plotted for the aforementioned values of the cut-off energy. In the same figure, we plotted three lines as reference for some peculiar parametrisation of $\widetilde{g}$ already mentioned in Section II: it is clear that for each one of these models the unitarity condition is always more important than the triviality one.

As for these specific parametrisations, in Figure 2 we have plotted the boundary value of g_1' against the evolution/cut-off scale Λ, using both the perturbative unitarity stability condition (dashed lines) and the triviality condition (dotted lines). For the $U(1)_{B-L}$ (Figure 2a) and the $U(1)_\chi$ (Figure 2b) extensions of the SM model, the unitarity bound is always more stringent than the triviality one. For the $U(1)_R$ (Figure 2c) extension, this is only true if $\Lambda > 10^{10}$ GeV. In Figure 2d we have plotted the best bound on g_1' (and then $\widetilde{g}$) against the evolution/cut-off energy scale Λ.

In order to summarise these results, in Table III we present a comparison between the triviality and the unitarity bounds on g_1' for several values of the energy scale Λ for our choice of Minimal Z' Models.

V. CONCLUSIONS

In this paper, we have shown that, by combining perturbative unitarity and RGE methods, one can significantly constrain the gauge couplings (g_1' and $\widetilde{g}$) of a generic/universal (non-

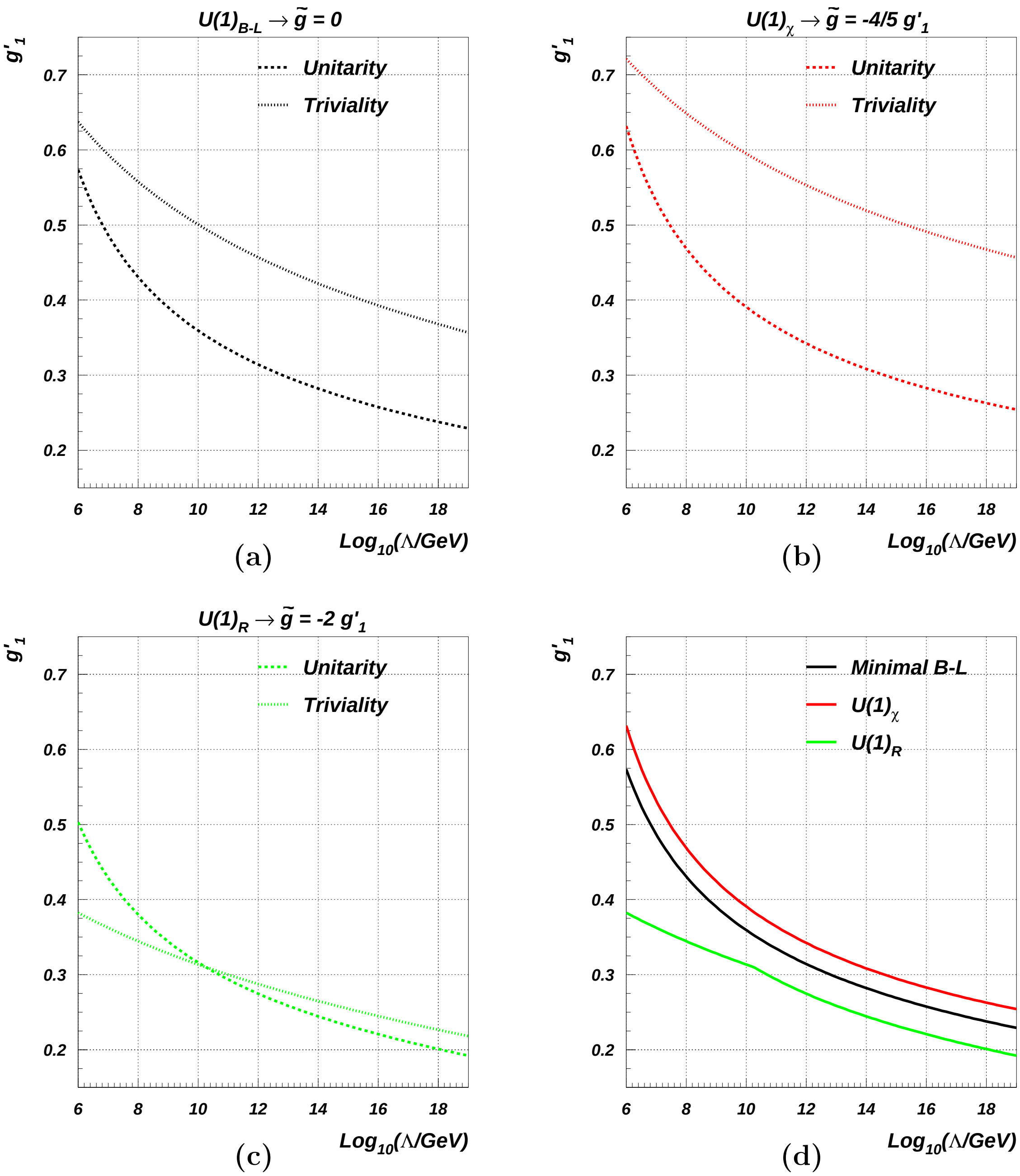

FIG. 2: The g'_1 ($\widetilde{g}$) couplings bounded either by triviality (dotted lines) or unitarity (dashed lines) conditions plotted against the evolution/cut-off energy for several peculiar choices of the gauge couplings parametrisation: $U(1)_{B-L}$ (black lines, $\widetilde{g}=0$: Figure 2a), $U(1)_\chi$ (dark red/grey lines, $\widetilde{g}=-4/5g'_1$: Figure 2b), $U(1)_R$ (light green/grey lines, $\widetilde{g}=-2g'_1$: Figure 2c). Figure 2d shows a summary of the most stringent bounds at different values of the considered parametrisations of $\widetilde{g}$ (Table I).

exotic/non-anomalous) Z' gauge boson, by imposing limits on their upper values that are more stringent than standard triviality bounds. (Also notice that, as unitarity is more constraining than triviality, the stability of the perturbative solutions obtained through the former is already guaranteed by the latter.)

The present work, alongside [18], [17] and [28], is in particular part of the long-term effort to establish the theoretical bounds on the parameter space of the $B-L$ based $U(1)$ extension of the SM.

Acknowledgements

GMP would like to thank Alexander S. Belyaev, Stephen F. King and Douglas A. Ross for helpful discussions. This work is supported in part by the NExT Institute.

Appendix A: Gauge-fixing Lagrangian of Minimal Z' Models

As for the Goldstone bosons sector, the mass matrix and interactions for the ghost fields are defined by the matrix $\mathcal{D}$, as in equation (24), via

$$m^2_{ghost} = \mathcal{D}(\mathcal{D})^T. \tag{A1}$$

Notice that the m^2_{ghost} and the m^2_v of equation (26) have different numbers of zero-eigenvalues, but their non-zero eigenvalues are in a one-to-one correspondence; furthermore, the eigenvalues of the gauge-fixing mass matrix are the same of the gauge boson mass matrix.

Then, the ghost Lagrangian is defined, in the t'Hooft-Feynman gauge, as

$$\mathcal{L}_{ghost} = -\bar{c}^a \left[(\partial_\mu D^\mu)^{ab} + \mathcal{D}^a \cdot \left(\mathcal{D}^b + \mathcal{S}^b \right)^T \right] c^b \,, \tag{A2}$$

where the matrix $\mathcal{S}$ represents the link between the fluctuations (Goldstones) of the Higgses around their $VEVs$ and the gauge bosons; a convenient way to write this matrix is

$$(\mathcal{S})^T = \begin{pmatrix} \frac{g}{2}h & \frac{g}{2}\phi_3 & -\frac{g}{2}\phi_2 & -\frac{g_1}{2}\phi_2 & -\frac{\widetilde{g}}{2}\phi_2 \\ -\frac{g}{2}\phi_3 & \frac{g}{2}h & \frac{g}{2}\phi_1 & \frac{g_1}{2}\phi_1 & \frac{\widetilde{g}}{2}\phi_1 \\ \frac{g}{2}\phi_2 & -\frac{g}{2}\phi_1 & \frac{g}{2}h & -\frac{g_1}{2}h & -\frac{\widetilde{g}}{2}h \\ 0 & 0 & 0 & 0 & -2h'g_1' \end{pmatrix} . \tag{A3}$$

Finally, the ghost fields ($\overset{(-)}{c}$) read as

$$c = \left(\, w_1^g \;\; w_2^g \;\; w_3^g \;\; B^g \;\; (B')^g \, \right) . \tag{A4}$$

Appendix B: Feynman rules associated with a neutral gauge boson exchange in a scalar two-body scattering

In the following we list the set of Feynman rules that enter in the calculation described Section III; these have been obtained by means of implementing the information of the scalar Lagrangian (see Section II and Appendix A) in the LanHEP package [29]; all the momenta p's are considered in-coming:

$$\begin{aligned} h_1 - Z - z \;:\Rightarrow\; & \frac{-1}{2c_W\ s_W}\big(s_W\ s_{\theta'}\ c_W\ c_\alpha\ c_{\alpha g}\ \widetilde{g}\ p_h^\mu - s_W\ s_{\theta'}\ c_W\ c_\alpha\ c_{\alpha g}\ \widetilde{g}\ p_z^\mu \\ + \;& c_\alpha\ c_{\alpha g}\ c_{\theta'}\ e\ p_z^\mu - c_\alpha\ c_{\alpha g}\ c_{\theta'}\ e\ p_h^\mu - 4s_W\ s_\alpha\ s_{\alpha g}\ s_{\theta'}\ c_W\ g_1'\ p_h^\mu \\ + \;& 4s_W\ s_\alpha\ s_{\alpha g}\ s_{\theta'}\ c_W\ g_1'\ p_z^\mu\big) \end{aligned} \tag{B1}$$

$$\begin{aligned} h_1 - Z - z' \;:\Rightarrow\; & \frac{1}{2c_W\ s_W}\big(s_W\ s_{\alpha g}\ s_{\theta'}\ c_W\ c_\alpha\ \widetilde{g}\ p_h^\mu - s_W\ s_{\alpha g}\ s_{\theta'}\ c_W\ c_\alpha\ \widetilde{g}\ p_{z'}^\mu \\ + \;& s_{\alpha g}\ c_\alpha\ c_{\theta'}\ e\ p_{z'}^\mu - s_{\alpha g}\ c_\alpha\ c_{\theta'}\ e\ p_h^\mu + 4s_W\ s_\alpha\ s_{\theta'}\ c_W\ c_{\alpha g}\ g_1'\ p_h^\mu \\ - \;& 4s_W\ s_\alpha\ s_{\theta'}\ c_W\ c_{\alpha g}\ g_1'\ p_{z'}^\mu\big) \end{aligned} \tag{B2}$$

$$\begin{aligned} h_1 - z - Z' \;:\Rightarrow\; & \frac{-1}{2c_W\ s_W}\big(s_W\ c_W\ c_\alpha\ c_{\alpha g}\ c_{\theta'}\ \widetilde{g}\ p_h^\mu - s_W\ c_W\ c_\alpha\ c_{\alpha g}\ c_{\theta'}\ \widetilde{g}\ p_z^\mu \\ - \;& s_{\theta'}\ c_\alpha\ c_{\alpha g}\ e\ p_z^\mu + s_{\theta'}\ c_\alpha\ c_{\alpha g}\ e\ p_h^\mu - 4s_W\ s_\alpha\ s_{\alpha g}\ c_W\ c_{\theta'}\ g_1'\ p_h^\mu \\ + \;& 4s_W\ s_\alpha\ s_{\alpha g}\ c_W\ c_{\theta'}\ g_1'\ p_z^\mu\big) \end{aligned} \tag{B3}$$

$$\begin{aligned} h_1 - Z' - z' \;:\Rightarrow\; & \frac{1}{2c_W\ s_W}\big(s_W\ s_{\alpha g}\ c_W\ c_\alpha\ c_{\theta'}\ \widetilde{g}\ p_h^\mu - s_W\ s_{\alpha g}\ c_W\ c_\alpha\ c_{\theta'}\ \widetilde{g}\ p_{z'}^\mu \\ - \;& s_{\alpha g}\ s_{\theta'}\ c_\alpha\ e\ p_{z'}^\mu + s_{\alpha g}\ s_{\theta'}\ c_\alpha\ e\ p_h^\mu + 4s_W\ s_\alpha\ c_W\ c_{\alpha g}\ c_{\theta'}\ g_1'\ p_h^\mu \\ - \;& 4s_W\ s_\alpha\ c_W\ c_{\alpha g}\ c_{\theta'}\ g_1'\ p_{z'}^\mu\big) \end{aligned} \tag{B4}$$

$$\begin{aligned} h_2 - Z - z \;:\Rightarrow\; & \frac{-1}{2c_W\ s_W}\big(s_W\ s_\alpha\ s_{\theta'}\ c_W\ c_{\alpha g}\ \widetilde{g}\ p_h^\mu - s_W\ s_\alpha\ s_{\theta'}\ c_W\ c_{\alpha g}\ \widetilde{g}\ p_z^\mu \\ + \;& s_\alpha\ c_{\alpha g}\ c_{\theta'}\ e\ p_z^\mu - s_\alpha\ c_{\alpha g}\ c_{\theta'}\ e\ p_h^\mu + 4s_W\ s_{\alpha g}\ s_{\theta'}\ c_W\ c_\alpha\ g_1'\ p_h^\mu \\ - \;& 4s_W\ s_{\alpha g}\ s_{\theta'}\ c_W\ c_\alpha\ g_1'\ p_z^\mu\big) \end{aligned} \tag{B5}$$

$$\begin{aligned} h_2 - Z - z' \;:\Rightarrow\; & \frac{1}{2c_W\ s_W}\big(s_W\ s_\alpha\ s_{\alpha g}\ s_{\theta'}\ c_W\ \widetilde{g}\ p_h^\mu - s_W\ s_\alpha\ s_{\alpha g}\ s_{\theta'}\ c_W\ \widetilde{g}\ p_{z'}^\mu \\ + \;& s_\alpha\ s_{\alpha g}\ c_{\theta'}\ e\ p_{z'}^\mu - s_\alpha\ s_{\alpha g}\ c_{\theta'}\ e\ p_h^\mu - 4s_W\ s_{\theta'}\ c_W\ c_\alpha\ c_{\alpha g}\ g_1'\ p_h^\mu \\ + \;& 4s_W\ s_{\theta'}\ c_W\ c_\alpha\ c_{\alpha g}\ g_1'\ p_{z'}^\mu\big) \end{aligned} \tag{B6}$$

$$
\begin{aligned}
h_2 - z - Z' \ :\Rightarrow\ & \frac{-1}{2c_W\ s_W}\big(s_W\ s_\alpha\ c_W\ c_{\alpha g}\ c_{\theta'}\ \widetilde{g}\ p_h^\mu - s_W\ s_\alpha\ c_W\ c_{\alpha g}\ c_{\theta'}\ \widetilde{g}\ p_z^\mu \\
- &\ \ s_\alpha\ s_{\theta'}\ c_{\alpha g}\ e\ p_z^\mu + s_\alpha\ s_{\theta'}\ c_{\alpha g}\ e\ p_h^\mu + 4s_W\ s_{\alpha g}\ c_W\ c_\alpha\ c_{\theta'}\ g_1'\ p_h^\mu \\
- &\ \ 4s_W\ s_{\alpha g}\ c_W\ c_\alpha\ c_{\theta'}\ g_1'\ p_z^\mu\big) \qquad \text{(B7)} \\
h_2 - Z' - z' \ :\Rightarrow\ & \frac{1}{2c_W\ s_W}\big(s_W\ s_\alpha\ s_{\alpha g}\ c_W\ c_{\theta'}\ \widetilde{g}\ p_h^\mu - s_W\ s_\alpha\ s_{\alpha g}\ c_W\ c_{\theta'}\ \widetilde{g}\ p_{z'}^\mu \\
- &\ \ s_\alpha\ s_{\alpha g}\ s_{\theta'}\ e\ p_{z'}^\mu + s_\alpha\ s_{\alpha g}\ s_{\theta'}\ e\ p_h^\mu - 4s_W\ c_W\ c_\alpha\ c_{\alpha g}\ c_{\theta'}\ g_1'\ p_h^\mu \\
+ &\ \ 4s_W\ c_W\ c_\alpha\ c_{\alpha g}\ c_{\theta'}\ g_1'\ p_{z'}^\mu\big) \qquad \text{(B8)}
\end{aligned}
$$

$$
\begin{aligned}
w^+ - w^- - Z \ :\Rightarrow\ & \frac{i}{2c_W\ s_W}\big((1-2s_W^2)\ c_{\theta'}\ e\ p_{w^-}^\mu + s_W\ s_{\theta'}\ c_W\ \widetilde{g}\ p_{w^-}^\mu \\
- &\ \ (1-2{s_W}^2)\ c_{\theta'}\ e\ p_{w^+}^\mu - s_W\ s_{\theta'}\ c_W\ \widetilde{g}\ p_{w^+}^\mu\big) \qquad \text{(B9)} \\
w^+ - w^- - Z' \ :\Rightarrow\ & \frac{-i}{2c_W\ s_W}\big((1-2s_W^2)\ s_{\theta'}\ e\ p_{w^-}^\mu - s_W\ c_W\ c_{\theta'}\ \widetilde{g}\ p_{w^-}^\mu \\
- &\ \ (1-2s_W^2)\ s_{\theta'}\ e\ p_{w^+}^\mu + s_W\ c_W\ c_{\theta'}\ \widetilde{g}\ p_{w^+}^\mu\big) \qquad \text{(B10)}
\end{aligned}
$$

In the previous formulae, we have used the following notation:

$$
\begin{aligned}
c_W(s_W) &\rightarrow \cos\theta_W(\sin\theta_W), \\
c_\alpha(s_\alpha) &\rightarrow \cos\alpha(\sin\alpha), \\
c_{\alpha g}(s_{\alpha g}) &\rightarrow \cos\alpha_g(\sin\alpha_g), \qquad \text{(B11)} \\
c_{\theta'}(s_{\theta'}) &\rightarrow \cos\theta'(\sin\theta'), \\
e &\rightarrow \frac{gg_1}{\sqrt{g^2+g_1^2}}.
\end{aligned}
$$

Appendix C: Explicit value of the $f^{z,z'}_{i,j}$ coefficients of equations (33)-(34)

The non-vanishing coefficients related to the structure of equation (34), in the high energy limit, for each entry of the scattering matrix are the following:

$$
\begin{aligned}
& f^{z}_{zz,h_1h_1} = \\
= \; & \frac{1}{4}\left(-c_\alpha^2 c_{\alpha g}^2 c_{\theta'}^2\left(g^2+g_1^2\right)+(c_\alpha c_{\alpha g}\widetilde{g}-4g_1' s_\alpha s_{\alpha g})^2 s_{\theta'}^2\right), \qquad \text{(C1)}
\end{aligned}
$$

$$
\begin{aligned}
& f^{z'}_{zz,h_1h_1} = \\
= \; & \frac{1}{4}\left(16c_{\theta'}^2(g_1')^2 s_\alpha^2 s_{\alpha g}^2 - 8c_\alpha c_{\alpha g} c_{\theta'} g_1' s_\alpha s_{\alpha g}\left(c_{\theta'}\widetilde{g}+\sqrt{g^2+g_1^2}s_{\theta'}\right)\right. \\
+ \; & \left. c_\alpha^2 c_{\alpha g}^2\left(c_{\theta'}^2\widetilde{g}^2+2c_{\theta'}\sqrt{g^2+g_1^2}\widetilde{g}s_{\theta'}+\left(g^2+g_1^2\right)s_{\theta'}^2\right)\right), \qquad \text{(C2)}
\end{aligned}
$$

$$
\begin{aligned}
& f^{z}_{zz,h_1h_2} = \\
= \; & \frac{1}{4}\left(4c_{\alpha g} g_1' s_\alpha^2 s_{\alpha g} s_{\theta'}\left(c_{\theta'}\sqrt{g^2+g_1^2}-\widetilde{g}s_{\theta'}\right)\right. \\
+ \; & 4c_\alpha^2 c_{\alpha g} g_1' s_{\alpha g} s_{\theta'}\left(-c_{\theta'}\sqrt{g^2+g_1^2}+\widetilde{g}s_{\theta'}\right) \\
+ \; & \left. c_\alpha s_\alpha\left(-16(g_1')^2 s_{\alpha g}^2 s_{\theta'}^2 + c_{\alpha g}^2\left(c_{\theta'}^2\left(g^2+g_1^2\right)-2c_{\theta'}\sqrt{g^2+g_1^2}\widetilde{g}s_{\theta'}+\widetilde{g}^2 s_{\theta'}^2\right)\right)\right), \qquad \text{(C3)}
\end{aligned}
$$

$$
\begin{aligned}
& f^{z'}_{zz,h_1h_2} = \\
= \; & \frac{1}{4}\left(4c_\alpha^2 c_{\alpha g} c_{\theta'} g_1' s_{\alpha g}\left(c_{\theta'}\widetilde{g}+\sqrt{g^2+g_1^2}s_{\theta'}\right)\right. \\
- \; & 4c_{\alpha g} c_{\theta'} g_1' s_\alpha^2 s_{\alpha g}\left(c_{\theta'}\widetilde{g}+\sqrt{g^2+g_1^2}s_{\theta'}\right) \\
+ \; & \left. c_\alpha s_\alpha\left(-16c_{\theta'}^2(g_1')^2 s_{\alpha g}^2 + c_{\alpha g}^2\left(c_{\theta'}^2\widetilde{g}^2+2c_{\theta'}\sqrt{g^2+g_1^2}\widetilde{g}s_{\theta'}+\left(g^2+g_1^2\right)s_{\theta'}^2\right)\right)\right), \qquad \text{(C4)}
\end{aligned}
$$

$$
\begin{aligned}
& f^{z}_{zz,h_2h_2} = \\
= \; & \frac{1}{4}\left(c_{\alpha g}^2 c_{\theta'}^2\left(g^2+g_1^2\right)s_\alpha^2 - 2c_{\alpha g}^2 c_{\theta'}\sqrt{g^2+g_1^2}\widetilde{g}s_\alpha^2 s_{\theta'}\right. \\
- \; & \left. 8c_\alpha c_{\alpha g} c_{\theta'}\sqrt{g^2+g_1^2}g_1' s_\alpha s_{\alpha g} s_{\theta'} + (c_{\alpha g}\widetilde{g}s_\alpha + 4c_\alpha g_1' s_{\alpha g})^2 s_{\theta'}^2\right) \qquad \text{(C5)}
\end{aligned}
$$

$$
\begin{aligned}
& f^{z'}_{zz,h_2h_2} = \\
= \; & \frac{1}{4}\left(c_{\theta'}^2(c_{\alpha g}\widetilde{g}s_\alpha+4c_\alpha g_1' s_{\alpha g})^2 - c_{\alpha g}^2\left(g^2+g_1^2\right)s_\alpha^2 s_{\theta'}^2\right), \qquad \text{(C6)}
\end{aligned}
$$

$$
\begin{aligned}
& f^{z}_{zz',h_1h_1} = \\
= \;& \frac{1}{4}\left(16c_{\alpha g}(g_1')^2 s_\alpha^2 s_{\alpha g} s_{\theta'}^2 + 4c_\alpha g_1' s_\alpha \left(c_{\alpha g}^2 + s_{\alpha g}^2\right) s_{\theta'} \left(c_{\theta'}\sqrt{g^2+g_1^2} - \widetilde{g} s_{\theta'}\right)\right. \\
+ \;& \left. c_\alpha^2 c_{\alpha g} s_{\alpha g}\left(c_{\theta'}^2\left(g^2+g_1^2\right) - 2c_{\theta'}\sqrt{g^2+g_1^2}\,\widetilde{g} s_{\theta'} + \widetilde{g}^2 s_{\theta'}^2\right)\right),
\end{aligned} \tag{C7}
$$

$$
\begin{aligned}
& f^{z'}_{zz',h_1h_1} = \\
= \;& \frac{1}{4}\left(16c_{\alpha g} c_{\theta'}^2 (g_1')^2 s_\alpha^2 s_{\alpha g} - 4c_\alpha c_{\theta'} g_1' s_\alpha \left(c_{\alpha g}^2 + s_{\alpha g}^2\right)\left(c_{\theta'}\widetilde{g} + \sqrt{g^2+g_1^2}\, s_{\theta'}\right)\right. \\
+ \;& \left. c_\alpha^2 c_{\alpha g} s_{\alpha g}\left(c_{\theta'}^2 \widetilde{g}^2 + 2c_{\theta'}\sqrt{g^2+g_1^2}\,\widetilde{g} s_{\theta'} + \left(g^2+g_1^2\right) s_{\theta'}^2\right)\right),
\end{aligned} \tag{C8}
$$

$$
\begin{aligned}
& f^{z}_{zz',h_1h_2} = \\
= \;& \frac{1}{4}\left(4g_1' s_\alpha^2 s_{\alpha g}^2 s_{\theta'}\left(c_{\theta'}\sqrt{g^2+g_1^2} - \widetilde{g} s_{\theta'}\right) + 4c_\alpha^2 c_{\alpha g}^2 g_1' s_{\theta'}\left(-c_{\theta'}\sqrt{g^2+g_1^2} + \widetilde{g} s_{\theta'}\right)\right. \\
+ \;& \left. c_\alpha c_{\alpha g} s_\alpha s_{\alpha g}\left(c_{\theta'}^2\left(g^2+g_1^2\right) - 2c_{\theta'}\sqrt{g^2+g_1^2}\,\widetilde{g} s_{\theta'} + \left(-16(g_1')^2 + \widetilde{g}^2\right) s_{\theta'}^2\right)\right),
\end{aligned} \tag{C9}
$$

$$
\begin{aligned}
& f^{z'}_{zz',h_1h_2} = \\
= \;& \frac{1}{4}\left(4c_\alpha^2 c_{\alpha g}^2 c_{\theta'} g_1'\left(c_{\theta'}\widetilde{g} + \sqrt{g^2+g_1^2}\, s_{\theta'}\right) - 4c_{\theta'} g_1' s_\alpha^2 s_{\alpha g}^2\left(c_{\theta'}\widetilde{g} + \sqrt{g^2+g_1^2}\, s_{\theta'}\right)\right. \\
+ \;& \left. c_\alpha c_{\alpha g} s_\alpha s_{\alpha g}\left(c_{\theta'}^2\left(-16(g_1')^2 + \widetilde{g}^2\right) + 2c_{\theta'}\sqrt{g^2+g_1^2}\,\widetilde{g} s_{\theta'} + \left(g^2+g_1^2\right) s_{\theta'}^2\right)\right),
\end{aligned} \tag{C10}
$$

$$
\begin{aligned}
& f^{z}_{zz',h_2h_2} = \\
= \;& \frac{1}{4}\left(4c_\alpha g_1' s_\alpha s_{\alpha g}^2 s_{\theta'}\left(c_{\theta'}\sqrt{g^2+g_1^2} - \widetilde{g} s_{\theta'}\right) - 4c_\alpha c_{\alpha g}^2 g_1' s_\alpha s_{\theta'}\left(c_{\theta'}\sqrt{g^2+g_1^2} - \widetilde{g} s_{\theta'}\right)\right. \\
+ \;& \left. c_{\alpha g} s_{\alpha g}\left(c_{\theta'}^2\left(g^2+g_1^2\right) s_\alpha^2 - 2c_{\theta'}\sqrt{g^2+g_1^2}\,\widetilde{g} s_\alpha^2 s_{\theta'} + \left(-16c_\alpha^2(g_1')^2 + \widetilde{g}^2 s_\alpha^2\right) s_{\theta'}^2\right)\right),
\end{aligned} \tag{C11}
$$

$$
\begin{aligned}
& f^{z'}_{zz',h_2h_2} = \\
= \;& \frac{1}{4}\left(-16c_\alpha^2 c_{\alpha g} c_{\theta'}^2 (g_1')^2 s_{\alpha g} + 4c_\alpha c_{\theta'} g_1' s_\alpha (c_{\alpha g}^2 - s_{\alpha g}^2)\left(c_{\theta'}\widetilde{g} + \sqrt{g^2+g_1^2}\, s_{\theta'}\right)\right. \\
+ \;& \left. c_{\alpha g} s_\alpha^2 s_{\alpha g}\left(c_{\theta'}^2 \widetilde{g}^2 + 2c_{\theta'}\sqrt{g^2+g_1^2}\,\widetilde{g} s_{\theta'} + \left(g^2+g_1^2\right) s_{\theta'}^2\right)\right),
\end{aligned} \tag{C12}
$$

$$
\begin{aligned}
& f^{z}_{z'z',h_1h_1} = \\
= \; & \frac{1}{4}\Bigg(c_\alpha^2 c_{\theta'}^2\left(g^2+g_1^2\right)s_{\alpha g}^2 + 8c_\alpha c_{\alpha g} c_{\theta'}\sqrt{g^2+g_1^2}\,g_1' s_\alpha s_{\alpha g} s_{\theta'} \\
- \; & 2c_\alpha^2 c_{\theta'}\sqrt{g^2+g_1^2}\,\widetilde{g} s_{\alpha g}^2 s_{\theta'} + (-4c_{\alpha g} g_1' s_\alpha + c_\alpha \widetilde{g} s_{\alpha g})^2 s_{\theta'}^2\Bigg)\,, \qquad \text{(C13)}
\end{aligned}
$$

$$
\begin{aligned}
& f^{z'}_{z'z',h_1h_1} = \\
= \; & \frac{1}{4}\Bigg(16c_{\alpha g}^2 c_{\theta'}^2 (g_1')^2 s_\alpha^2 - 8c_\alpha c_{\alpha g} c_{\theta'} g_1' s_\alpha s_{\alpha g}\left(c_{\theta'}\widetilde{g} + \sqrt{g^2+g_1^2}\,s_{\theta'}\right) \\
+ \; & c_\alpha^2 s_{\alpha g}^2\left(c_{\theta'}^2\widetilde{g}^2 + 2c_{\theta'}\sqrt{g^2+g_1^2}\,\widetilde{g} s_{\theta'} + \left(g^2+g_1^2\right)s_{\theta'}^2\right)\Bigg)\,, \qquad \text{(C14)}
\end{aligned}
$$

$$
\begin{aligned}
= \; & f^{z}_{z'z',h_1h_2} = \\
= \; & \frac{1}{4}\Bigg(4c_{\alpha g} g_1' s_\alpha^2 s_{\alpha g} s_{\theta'}\left(c_{\theta'}\sqrt{g^2+g_1^2} - \widetilde{g} s_{\theta'}\right) \\
+ \; & 4c_\alpha^2 c_{\alpha g} g_1' s_{\alpha g} s_{\theta'}\left(-c_{\theta'}\sqrt{g^2+g_1^2} + \widetilde{g} s_{\theta'}\right) \\
+ \; & c_\alpha s_\alpha\left(c_{\theta'}^2\left(g^2+g_1^2\right)s_{\alpha g}^2 - 2c_{\theta'}\sqrt{g^2+g_1^2}\,\widetilde{g} s_{\alpha g}^2 s_{\theta'} - \left(16c_{\alpha g}^2 (g_1')^2 - \widetilde{g}^2 s_{\alpha g}^2\right)s_{\theta'}^2\right)\Bigg)\,, \qquad \text{(C15)}
\end{aligned}
$$

$$
\begin{aligned}
& f^{z'}_{z'z',h_1h_2} = \\
= \; & \frac{1}{4}\Bigg(4c_\alpha^2 c_{\alpha g} c_{\theta'} g_1' s_{\alpha g}\left(c_{\theta'}\widetilde{g} + \sqrt{g^2+g_1^2}\,s_{\theta'}\right) \\
- \; & 4c_{\alpha g} c_{\theta'} g_1' s_\alpha^2 s_{\alpha g}\left(c_{\theta'}\widetilde{g} + \sqrt{g^2+g_1^2}\,s_{\theta'}\right) \\
+ \; & c_\alpha s_\alpha\left(-16c_{\alpha g}^2 c_{\theta'}^2 (g_1')^2 + s_{\alpha g}^2\left(c_{\theta'}^2\widetilde{g}^2 + 2c_{\theta'}\sqrt{g^2+g_1^2}\,\widetilde{g} s_{\theta'} + \left(g^2+g_1^2\right)s_{\theta'}^2\right)\right)\Bigg)\,, \qquad \text{(C16)}
\end{aligned}
$$

$$
\begin{aligned}
& f^{z}_{z'z',h_2h_2} = \\
= \; & \frac{1}{4}\Bigg(c_{\theta'}^2\left(g^2+g_1^2\right)s_\alpha^2 s_{\alpha g}^2 - 2c_{\theta'}\sqrt{g^2+g_1^2}\,s_\alpha s_{\alpha g}(4c_\alpha c_{\alpha g} g_1' + \widetilde{g} s_\alpha s_{\alpha g}) s_{\theta'} \\
+ \; & (4c_\alpha c_{\alpha g} g_1' + \widetilde{g} s_\alpha s_{\alpha g})^2 s_{\theta'}^2\Bigg)\,, \qquad \text{(C17)}
\end{aligned}
$$

$$
\begin{aligned}
& f^{z'}_{z'z',h_2h_2} = \\
= \; & \frac{1}{4}\Bigg(16c_\alpha^2 c_{\alpha g}^2 c_{\theta'}^2 (g_1')^2 + 8c_\alpha c_{\alpha g} c_{\theta'} g_1' s_\alpha s_{\alpha g}\left(c_{\theta'}\widetilde{g} + \sqrt{g^2+g_1^2}\,s_{\theta'}\right) \\
+ \; & s_\alpha^2 s_{\alpha g}^2\left(c_{\theta'}^2\widetilde{g}^2 + 2c_{\theta'}\sqrt{g^2+g_1^2}\,\widetilde{g} s_{\theta'} + \left(g^2+g_1^2\right)s_{\theta'}^2\right)\Bigg)\,. \qquad \text{(C18)}
\end{aligned}
$$

The non-vanishing coefficients related to the structure of equation (33), in the high energy

limit, are the following:

$$f^{z}_{w^+w^-} = \\ = \frac{c^2_{\theta'}\left(g^2-g_1^2\right)^2\sqrt{g^2+g_1^2}+2c_{\theta'}\left(g^4-g_1^4\right)\widetilde{g}s_{\theta'}+\left(g^2+g_1^2\right)^{3/2}\widetilde{g}^2 s^2_{\theta'}}{4\left(g^2+g_1^2\right)^{3/2}}, \tag{C19}$$

$$f^{z'}_{w^+w^-} = \\ = \frac{c^2_{\theta'}\left(g^2+g_1^2\right)^{3/2}\widetilde{g}^2-2c_{\theta'}\left(g^4-g_1^4\right)\widetilde{g}s_{\theta'}+\left(g^2-g_1^2\right)^2\sqrt{g^2+g_1^2}s^2_{\theta'}}{4\left(g^2+g_1^2\right)^{3/2}}. \tag{C20}$$

In the previous equations, we have used the notation of equations (B11).

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